Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2019

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

JMATHE2

This question paper consists of 14 pages, including an information sheet and a special answer book of 19 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of TEN (10) questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

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QUESTION 1

Eastern High School compared the Term 1 percentages of 20 Grade 12 learners consisting of 10 boys and 10 girls. The following data was recorded:

Boys' marks	41	30	24	65	72	15	83	52	60	38

Girls' mean mark = 51 Girls' standard deviation = 15,95

1.1 Calculate the mean mark for boys. (1)

1.2 Calculate the standard deviation for the boys' marks. (2)

1.3 Did boys or girls perform better? Give a reason for your answer. (2)

1.4 By what percentage must each of the boys' marks be adjusted so that the mean of boys can be the same as that of the girls? (1)

1.5 Will the boys' standard deviation increase, decrease or remain the same after the adjustment in QUESTION 1.4 above? (1)
[7]

QUESTION 2

The ages of people who were registering to vote at a voting station were recorded in the frequency table below:

Ages (in years)	Frequency	Cumulative Frequency
$18 \leq x < 28$		4
$28 \leq x < 38$		14
$38 \leq x < 48$		28
$48 \leq x < 58$	17	
$58 \leq x < 68$	12	
$68 \leq x < 78$	3	

2.1 Complete the frequency table. (2)

2.2 Draw the cumulative frequency graph (ogive). (3)

2.3 Write down the modal class. (1)

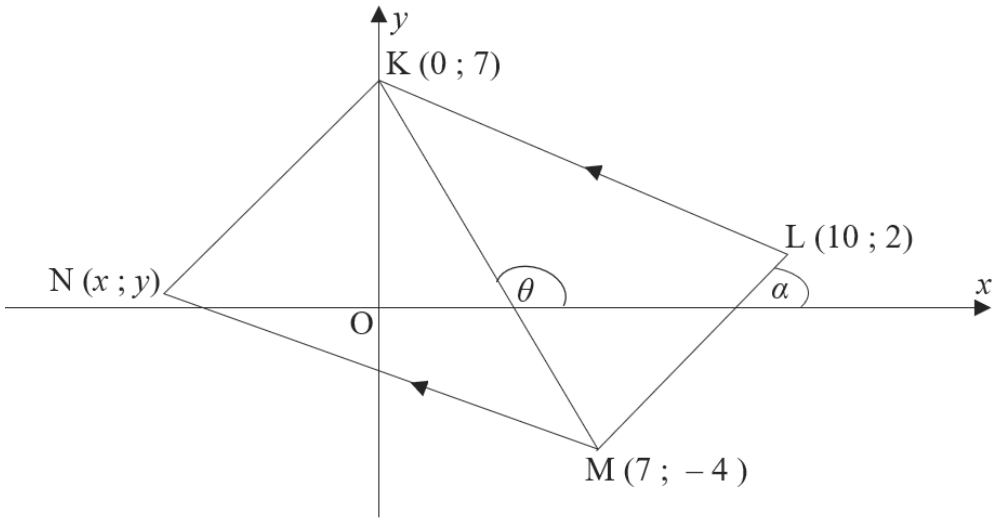
2.4 People who are 60 years and older are regarded as senior citizens and must not queue but be taken to the front. Estimate the number of senior citizens. (2)

2.5 Write down the lower (Q_1), middle (Q_2) and upper (Q_3) quartiles. (3)

2.6 Draw a box and whisker diagram to represent the ages of the voters. (2)
[13]

QUESTION 3

In the diagram below, K(0 ; 7), L(10 ; 2), M(7 ; – 4) and N(x ; y) are vertices of quadrilateral KLMN with MN || KL. θ and α are the angles formed by KM and ML with the x-axis respectively.



3.1 Determine:

3.1.1 The length of KL. Leave your answer in simplified surd form (2)

3.1.2 The gradient of KM (2)

3.1.3 The size of α, the angle of inclination of LM (3)

3.1.4 The size of $\hat{LMK}$ (4)

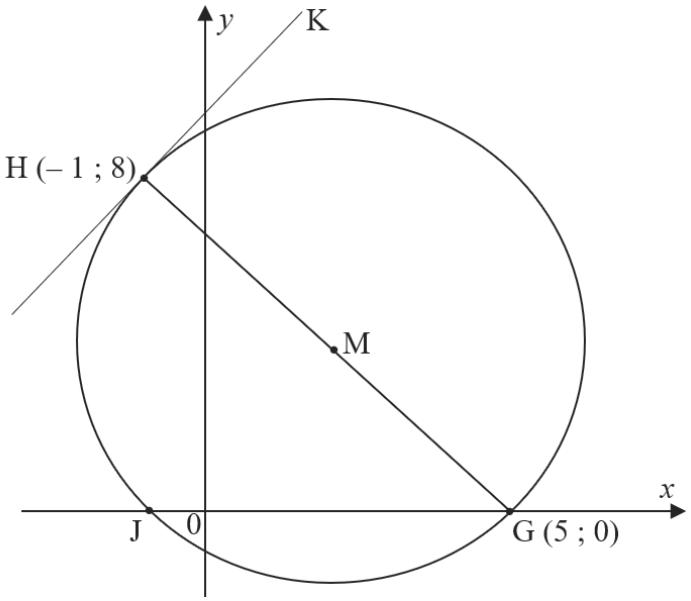
3.2 Determine the coordinates of N if KLMN is a parallelogram.
Show ALL calculations. (4)

3.3 Is $\hat{LMN}$ a right angle or not? Justify your answer by calculation(s). (2)

3.4 Determine the area of ΔKNM. (5)
[22]

QUESTION 4

In the diagram below, circle with centre M, diameter GH with G(5 ; 0) and tangent HK with point of contact at H(– 1 ; 8) is given.



4.1 Write down the coordinates of M. (2)

4.2 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.3 Determine the equation of the tangent HK. (4)

4.4 Determine the coordinates of J. (3)

4.5 Find the new coordinates of J if the circle is rotated 180° around the centre M. (2)

4.6 The equation of another circle is given as $x^2+y^2-12x-2y+17=0$.
Does the centre of the new circle lie on, inside or outside the originally given circle? Justify your answer with relevant calculations. (5)
[19]

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QUESTION 5

5.1 If $\sin 42° = k$, determine the following in terms of k.

5.1.1 $\tan 42°$ (2)

5.1.2 $\sin 84°$ (3)

5.1.3 $\sin 3°$ (4)

5.2 Simplify to a single trigonometric ratio:

$$\frac{\sin(x-450°).\tan(180°+x).\sin(90°-x)}{\cos(-x)}$$ (6)

5.3 Consider the identity: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

5.3.1 Complete: cos (A + B) = … (1)

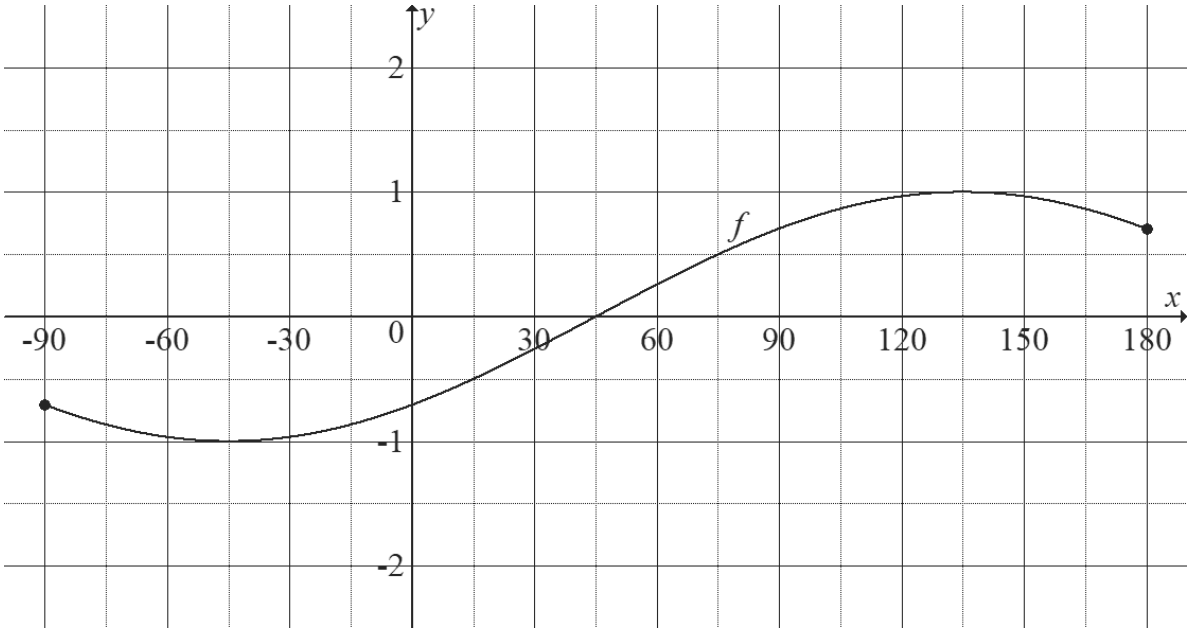
5.3.2 Prove the identity: $cos\,3\theta = 4\,cos^3\theta - 3\,cos\,\theta$ (4)

5.4 If $\cos\theta = 2p$ and $\cos 2\theta = 7p$, determine the possible value(s) of p. (5)

[25]

QUESTION 6

Given below is the graph of $f(x) = \sin(x - 45°)$, for $x \in [-90° ; 180°]$.



6.1 Write down the range of f. (1)

6.2 On the same set of axes, sketch the graph of $g(x) = \tan x$ for $x \in [-90° ; 180°]$ in the SPECIAL ANSWER BOOK. Show ALL intercepts with the axes as well as asymptotes and end points. (3)

6.3 Write down the period of g. (1)

6.4 Write down the value(s) of x for which $f(x) = g(x)$ for $x \in [-90° ; 180°]$. (1)

6.5 For which value(s) of x is $f(x).g(x) \geq 0$ for $x \in [0° ; 180°]$? (2)

6.6 Write down the equation of $h(x)$ if $h(x)$ is a result of shifting $f(x)$ such that its minimum value is zero. (1)
[9]

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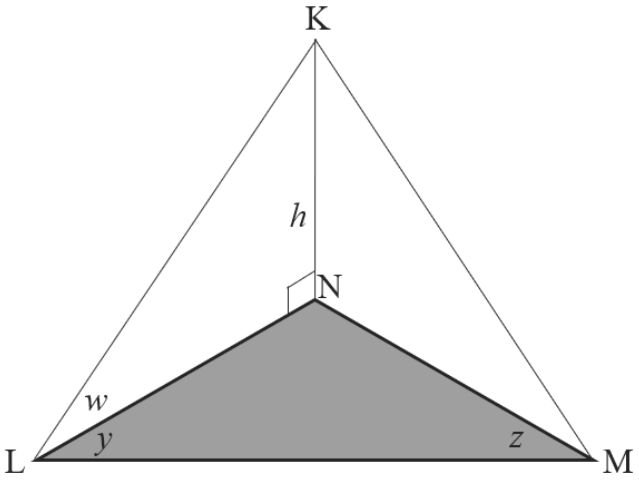
QUESTION 7

In the diagram KN represents a vertical tower, of height h metres, standing on a horizontal plane LMN. The angle of elevation of K, as seen from L, is w. $\hat{NLM} = y$ and $\hat{NML} = z$. (**NOTE**: all angles are measured in degrees).



7.1 Show that $LN = \dfrac{h}{\tan w}$ (1)

7.2 Hence, prove that $LM = \dfrac{h\sin(y+z)}{\tan w \sin z}$ (4)

7.3 Calculate LM if $h = 38\,\text{m}$, $w = 21^\circ$, $y = 52^\circ$ and $z = 59^\circ$. (2)

[7]

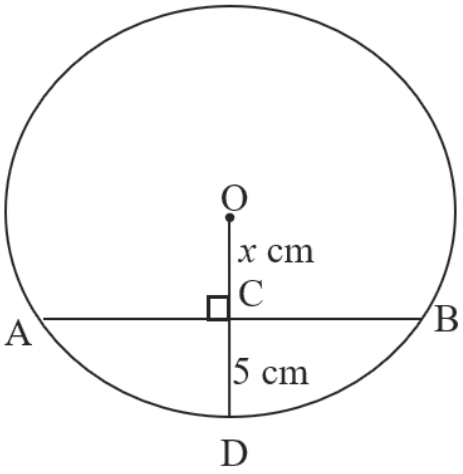
Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 Complete:
The perpendicular bisector of a chord passes through … (1)

8.2 In the diagram below, O is the centre of the circle, AB is a chord and OC ⊥ AB.

OC produced, intersects the circle at D. AB = 20 cm, CD = 5 cm and OC= x cm.



Determine, stating reasons:

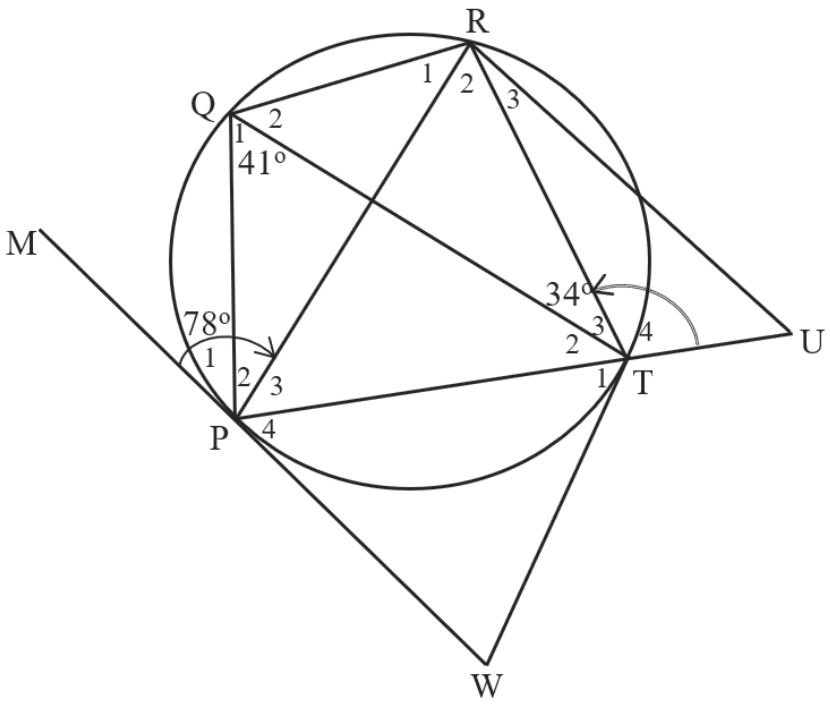
8.2.1 The length of AC (2)

8.2.2 The radius of the circle (4)
[7]

QUESTION 9

9.1 Complete:
Exterior angle of a cyclic quadrilateral is equal to … (1)

9.2 In the diagram below, points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U. Furthermore, $\hat{MPR} = 78°$, $\hat{PQT} = 41°$ and $\hat{QTR} = 34°$.



9.2.1 Write down, with reasons, THREE other angles that are each equal to 41°. (6)

9.2.2 Determine the following, stating reasons:

(a) $\hat{T}_2$ (2)

(b) $\hat{Q}_2$ (2)

(c) $\hat{T}_4$ (2)

(d) $\hat{W}$ (2)

9.2.3 Determine, with reasons, whether:

(a) QR || PT or not (2)

(b) PRTW is a cyclic quadrilateral or not (2)

(c) TQ is a diameter or not (2)

[21]

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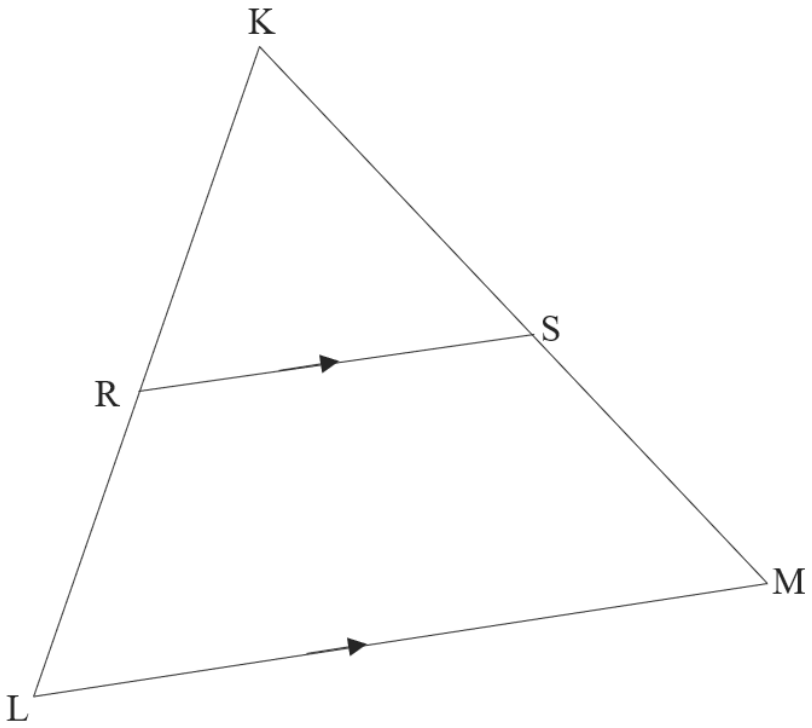
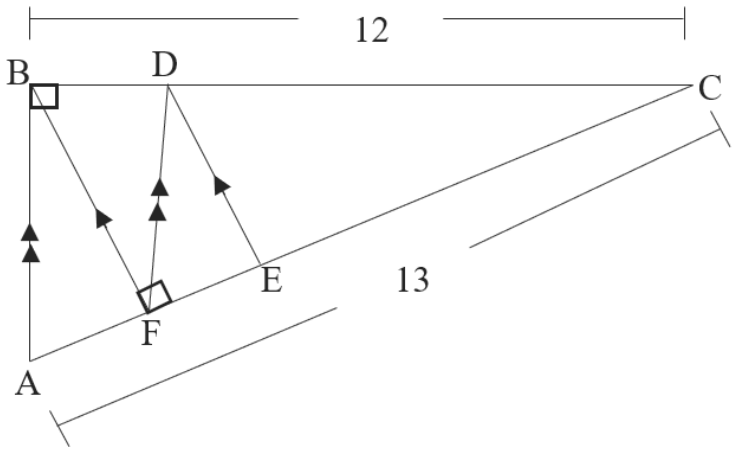
QUESTION 10

10.1 In the diagram below, ΔKLM is given with R and S on KL and KM respectively such that RS || LM.



Prove the theorem which states that $\frac{KR}{RL} = \frac{KS}{SM}$ (5)

10.2 In the diagram below, ΔABC is drawn with D on BC and F and E on AC such that AB || FD, BF || DE, AB ⊥ BC and BF ⊥ CA. Furthermore, CA = 13 units and CB = 12 units.



10.2.1 Write down the length of AB (1)

10.2.2 Prove, stating reasons, that:

(a) ΔCBA ||| ΔCFB (3)

(b) $CF = \frac{CB^2}{CA}$ (3)

10.2.3 Hence, determine the length of CF, correct to the nearest unit. (2)

10.2.4 Give the length of AF. (1)

10.2.5 Determine the length of FE.
Leave your answer in the form $\frac{a}{b}$. (5)
[20]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\dfrac{a}{\sin \mathrm{A}} = \dfrac{b}{\sin \mathrm{B}} = \dfrac{c}{\sin \mathrm{C}}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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