



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

**MATHEMATICS P3/WISKUNDE V3
FEBRUARY/MARCH/FEBRUARIE/MAART 2014**

MEMORANDUM

MARKS/PUNTE: 100

**This memorandum consists of 11 pages.
Hierdie memorandum bestaan uit 11 bladsye.**

NOTE:

- If a candidate answers a question TWICE, only mark the first attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out question.
- Consistent accuracy applies in ALL aspects of the marking memorandum.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE keer beantwoord, merk net die eerste poging.*
- *As 'n kandidaat 'n antwoord deurhaal en nie oordoen nie, merk die deurgehaalde antwoord.*
- *Volgehoue akkuraatheid moet DEURGAANS in die memorandum toegepas word.*

QUESTION/VRAAG 1

| | | | | | | | | | | | | | | | |
|----------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Temperature (°C) (x) | 17 | 15 | 13 | 16 | 11 | 13 | 10 | 8 | 6 | 7 | 8 | 4 | 5 | 9 | 6 |
| DO (ppm) (y) | 8 | 9 | 11 | 10 | 14 | 11 | 14 | 14 | 16 | 13 | 14 | 17 | 15 | 13 | 16 |

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|------------------------------|---|--|
| <p>1.1 & 1.3</p> | <p style="text-align: center;">Scatter plot showing the relationship between concentration of dissolved oxygen at various temperatures</p> | <p>1.1 ✓ 1 – 4 points correct ✓ 5 – 9 points correct ✓ all 12 points correct (3)</p> <p>1.3 ✓ correct gradient ✓ passing close to (8 ; 14) and (13 ; 11) (2)</p> |
| <p>1.2</p> | <p>$a = 19,01$ (19,00889878...) $b = -0,61$ (-0,6090100111...) $y = 19,01 - 0,61x$</p> | <p>✓✓ $a = 19,01$ ✓ $b = -0,61$ ✓ $y = 19,01 - 0,61x$ (4)</p> |

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| <p>1.4</p> | <p>$\hat{y} = 19,01 - 0,61(14)$ $= 10,47$</p> <p>OR If calculator is used $\hat{y} = 10,48$ (10,48275862...)</p> <p>OR If least squares method is used or the graph is used, $\hat{y} = 10,5$</p> | <p>✓ substitute $x = 14$ ✓ answer (2)</p> <p>✓✓ answer (2)</p> <p>✓✓ answer (2)</p> |
| <p>1.5</p> | <p>$r = -0,94$ (-0,9429488543...)</p> | <p>✓✓ answer (2)</p> |
| <p>1.6</p> | <p>There exists a very strong negative correlation between the variables. As the temperature in the lake water increases, so the concentration of dissolved oxygen decreases.</p> <p><i>Daar bestaan 'n sterk negatiewe korrelasie tussen die veranderlikes. Soos wat die temperatuur van die water in die meer verhoog, so verlaag die konsentrasie van die opgeloste suurstof.</i></p> | <p>✓ strong ✓ negative (2)</p> <p>[15]</p> |

QUESTION/VRAAG 2

| <p>2.1</p> | <p>$a = 73$ $b = 42$ $c = 107$ $d = 68$</p> <table border="1" data-bbox="172 1238 1026 1395"> <thead> <tr> <th></th> <th>Aged < 40</th> <th>Aged ≥ 40</th> <th>Totals</th> </tr> </thead> <tbody> <tr> <td>Liked the movie</td> <td>65</td> <td>37</td> <td>102</td> </tr> <tr> <td>Did not like the movie</td> <td>$b = 42$</td> <td>31</td> <td>$a = 73$</td> </tr> <tr> <td>Totals</td> <td>$c = 107$</td> <td>$d = 68$</td> <td>175</td> </tr> </tbody> </table> | | Aged < 40 | Aged ≥ 40 | Totals | Liked the movie | 65 | 37 | 102 | Did not like the movie | $b = 42$ | 31 | $a = 73$ | Totals | $c = 107$ | $d = 68$ | 175 | <p>✓ $a = 73$ ✓ $b = 42$ ✓ $c = 107$ ✓ $d = 68$ (4)</p> |
|------------------------|--|--|-----------|-----------|--------|-----------------|----|----|-----|------------------------|----------|----|----------|--------|-----------|----------|-----|---|
| | Aged < 40 | Aged ≥ 40 | Totals | | | | | | | | | | | | | | | |
| Liked the movie | 65 | 37 | 102 | | | | | | | | | | | | | | | |
| Did not like the movie | $b = 42$ | 31 | $a = 73$ | | | | | | | | | | | | | | | |
| Totals | $c = 107$ | $d = 68$ | 175 | | | | | | | | | | | | | | | |
| <p>2.2</p> | <p>$P(\text{less than 40 and did not like the movie}) = \frac{42}{175}$ (0,24)</p> | <p>✓ 42 ✓ 175 (2)</p> | | | | | | | | | | | | | | | | |
| <p>2.3</p> | <p>$P(\text{less than 40 and liked the movie}) = \frac{65}{175} = 0,37$ (0,3714285714...)</p> <p>$P(\text{Age less than 40}) = \frac{107}{175}$</p> <p>$P(\text{Critic liked the movie}) = \frac{102}{175}$</p> <p>$P(\text{Age less than 40}) \times P(\text{Critic liked the movie}) = \frac{107}{175} \times \frac{102}{175} = 0,36$ (0,3563755102...)</p> <p>Since $P(\text{less than 40 and liked the movie}) \neq P(\text{Age less than 40}) \times P(\text{Critic liked the movie})$, we can conclude that the events are not independent/nie onafhanklik nie.</p> | <p>✓ $P(\text{less than 40 and liked the movie}) = \frac{65}{175}$</p> <p>✓ $P(\text{Age less than 40}) = \frac{107}{175}$</p> <p>OR</p> <p>$P(\text{Critic liked the movie}) = \frac{102}{175}$</p> <p>✓ $P(\text{Age less than 40}) \times P(\text{Critic liked the movie}) = 0,36$</p> <p>✓ conclusion (4)</p> | | | | | | | | | | | | | | | | |

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| <p>OR</p> <p>$P(\text{less than 40 and did not like the movie}) = \frac{42}{175} = 0,24$</p> <p>$P(\text{Age less than 40}) = \frac{107}{175}$</p> <p>$P(\text{Critic did not like the movie}) = \frac{73}{175}$</p> <p>$P(\text{Age less than 40}) \times P(\text{Critic did not like the movie})$ $= \frac{107}{175} \times \frac{73}{175} = 0,255 \quad (0,2550530612\dots)$</p> <p>Since $P(\text{less than 40 and did not like the movie}) \neq P(\text{Age less than 40}) \times P(\text{Critic did not like the movie})$, we can conclude that the events are not independent/<i>nie onafhanklik nie</i>.</p> | <p>✓ $P(\text{less than 40 and did not like the movie}) = \frac{42}{175}$</p> <p>✓ $P(\text{Age less than 40}) = \frac{107}{175}$</p> <p>OR</p> <p>$P(\text{Critic did not like the movie}) = \frac{73}{175}$</p> <p>✓ $P(\text{Age less than 40}) \times P(\text{Critic liked the movie}) = 0,255$</p> <p>✓ conclusion</p> | <p>(4)</p> <p>[10]</p> |
|--|--|-------------------------------|

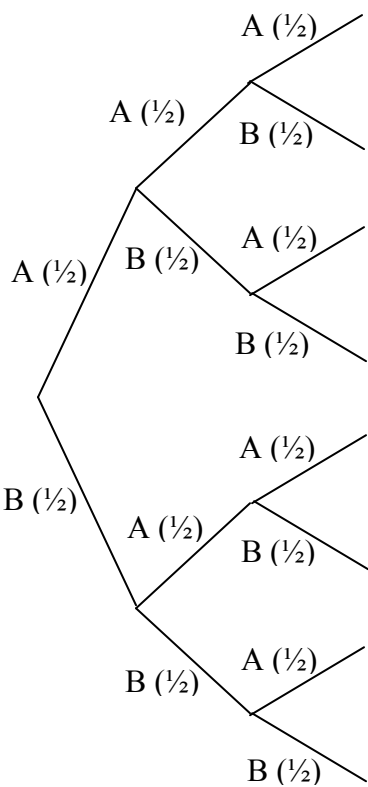
QUESTION/VRAAG 3

| | | |
|-----|---|---|
| 3.1 | <p>The interval 500 to 542 hours represents 48% of the data. $\Rightarrow 542$ is at 2 standard deviations to the right of the mean. $542 = 500 + 2\sigma$ $2\sigma = 42$ $\sigma = 21$</p> | <p>✓ 2 standard deviations</p> <p>✓ 21</p> <p style="text-align: right;">(2)</p> |
| 3.2 | <p>458 is at 2 standard deviations to the left of the mean. area between mean and 2 standard deviations = 48% 521 is at 1 standard deviation to the right of the mean. area between mean and 1 standard deviation = 34% \therefore Total area between 458 and 521 hours = 48% + 34% = 82%</p> | <p>✓ 48%</p> <p>✓ 34%</p> <p>✓ 82%</p> <p style="text-align: right;">(3)</p> |
| 3.3 | <p>The expected minimum lifetime will occur at 3 standard deviations to the left of the mean. \therefore Expected minimum lifetime = $500 - 3(21)\sigma = 437$ hours</p> | <p>✓ 3 standard deviations</p> <p>✓ 437</p> <p style="text-align: right;">(2)</p> <p>[7]</p> |

QUESTION/VRAAG 4

| | | | | | | | | | | | | | | | | | | | | | | |
|-----|--|--|--|--|--|--|--|--|--|--|--|--|---|--|--|--|--|--|--|--|--|--|
| 4.1 | Different ways for 8 learners to be seated $= 8!$ $= 40\,320$ | $\checkmark 8$ $\checkmark 40\,320$ (2) | | | | | | | | | | | | | | | | | | | | |
| 4.2 | Consider the 3 learners as a single entity. These 3 learners can be seated in $3! = 6$ different ways. Now this group of 3 and the remaining 5 learners can be seated in $6! = 720$ different ways. In total there are $6 \times 720 = 4320$ different ways for the 3 learners to be seated together. $3!6!$ $= 4\,320$ | $\checkmark 6$ $\checkmark 720$ $\checkmark 4\,320$ (3) | | | | | | | | | | | | | | | | | | | | |
| 4.3 | First let us consider the different number of ways that these 2 learners can be seated next to one another. This can be done in $2! \times 7! = 10\,080$ different ways. Now these two learners may not be seated next to one another in $40\,320 - 10\,080$ $= 30\,240$ different ways. OR/OF Let person A sit at the end of the row and person B not sit next to person A. This can be done in $1 \times 6 \times 6! \times 2$ different ways <table border="1" data-bbox="172 1424 983 1464"> <tr> <td>A</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> </table> <p style="text-align: center;">\uparrow not B</p> Let person A not sit at the end of the row. this can be done in $1 \times 6 \times 5 \times 5! \times 6$ different ways <table border="1" data-bbox="172 1693 983 1733"> <tr> <td></td><td>A</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> </table> <p style="text-align: center;">\uparrow \uparrow not B not B</p> In total we have $1 \times 6 \times 6! \times 2 + 1 \times 6 \times 5 \times 5! \times 6$ $= 30\,240$ different ways | A | | | | | | | | | | | A | | | | | | | | | $\checkmark 2! \times 7!$ or $10\,080$ $\checkmark 40\,320 - 10\,080$ \checkmark answer (3) $\checkmark 1 \times 6 \times 6! \times 2 = 8\,640$ $\checkmark 1 \times 6 \times 5 \times 5! \times 6 = 21\,600$ \checkmark answer (3) [8] |
| A | | | | | | | | | | | | | | | | | | | | | | |
| | A | | | | | | | | | | | | | | | | | | | | | |

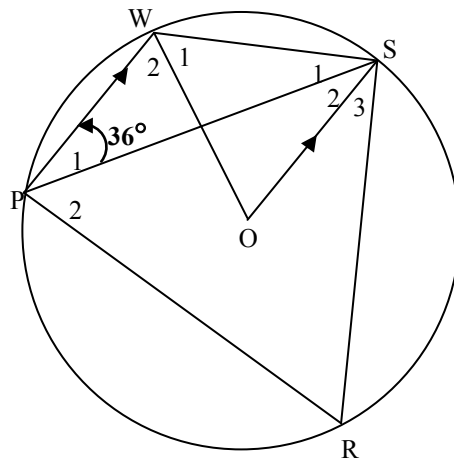
QUESTION/VRAAG 5

| | | |
|------------|--|--|
| <p>5.1</p> | <p>Let A represent Alfred winning a point and B represent Barry winning a point.</p> <p style="text-align: right;">Outcomes</p>  <p>(A ; A ; A) (A ; A ; B) (A ; B ; A) (A ; B ; B) (B ; A ; A) (B ; A ; B) (B ; B ; A) (B ; B ; B)</p> | <p>✓ first branch correct ✓ second branch correct ✓ third branch correct ✓ probabilities at each branch ✓ all outcomes listed (5)</p> |
| <p>5.2</p> | <p>$P(\text{Barry wins three points}) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$</p> | <p>✓ $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ ✓ $\frac{1}{8}$ (2)</p> |
| <p>5.3</p> | <p>$P(\text{Alfred wins two points and Barry wins one point})$ $= P(\text{A ; A ; B}) + P(\text{A ; B ; A}) + P(\text{B ; A ; A})$ $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{3}{8}$</p> | <p>✓ addition of $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ ✓ $\frac{3}{8}$ (2)</p> |
| <p>5.4</p> | <p>$P(\text{Alfred wins 3 of the four points})$ $= P(\text{AAAB}) + P(\text{AABA}) + P(\text{ABAA}) + P(\text{BAAA})$ $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= 4\left(\frac{1}{2}\right)^4$ $= \frac{1}{4}$</p> | <p>✓✓ $P(\text{AAAB}) + P(\text{AABA}) + P(\text{ABAA}) + P(\text{BAAA})$ ✓ $4\left(\frac{1}{2}\right)^4$ ✓ answer (4) [13]</p> |

QUESTION/VRAAG 6

| | | |
|-----|---|--|
| 6.1 | <p>By inspection</p> $T_{n+1} = T_n + T_{n-1} - 2; T_1 = 4, T_2 = 7; n \geq 1, n \in N$ $a = 1$ $b = -2$ <p>OR/OF</p> $T_{n+1} = T_n + aT_{n-1} + b$ $9 = 7 + 4a + b$ $2 = 4a + b \quad \dots (1)$ $14 = 9 + 7a + b$ $5 = 7a + b \quad \dots (2)$ $(2) - (1): \quad 3a = 3$ $a = 1$ $b = -2$ | <p>✓✓✓✓ Answer only: full marks</p> <p>(4)</p> <p>✓ $2 = 4a + b$</p> <p>✓ $5 = 7a + b$</p> <p>✓ $a = 1$</p> <p>✓ $b = -2$</p> <p>(4)</p> |
| 6.2 | $T_7 = 52$ | <p>✓ answer</p> <p>(1)</p> <p>[5]</p> |

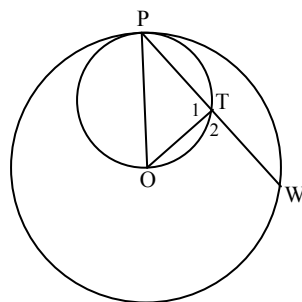
QUESTION/VRAAG 7



| | | |
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| 7.1 | $\hat{S}O\hat{W} = 72^\circ$ (\angle circ cent = 2 \angle circumference) (<i>middelpunts \angle = 2 omtreks \angle</i>) | ✓ $\hat{S}O\hat{W} = 72^\circ$ ✓ \angle circ cent = 2 \angle circumference (2) |
| 7.2 | $\hat{W}_2 = 72^\circ$ (alt \angle s; $PW \parallel SA$) / (<i>verw \anglee; $PW \parallel SA$</i>) | ✓ $\hat{W}_2 = 72^\circ$ ✓ $PW \parallel SO$ (2) |
| 7.3 | $\hat{O}S\hat{W} = \hat{W}_1$ (\angle s opp = radii)/(<i>\anglee teenoor = radiusse</i>) $2\hat{O}S\hat{W} + 72^\circ = 180^\circ$ (\angle sum Δ)/(<i>som van binne \anglee Δ</i>) $2\hat{O}S\hat{W} = 108^\circ$ $\hat{O}S\hat{W} = 54^\circ$ | ✓ $\hat{O}S\hat{W} = \hat{W}_1$ ✓ \angle s opp = radii ✓ answer (3) |
| 7.4 | $\hat{R} + \hat{W}_1 + \hat{W}_2 = 180^\circ$ (opp \angle s cyclic quad)/(<i>oorst \anglee koordevierhoek</i>) $\hat{R} + 54^\circ + 72^\circ = 180^\circ$ $\hat{R} = 54^\circ$ | ✓ $\hat{R} + \hat{W}_1 + \hat{W}_2 = 180^\circ$ ✓ opp \angle s cyclic quad ✓ answer (3) |

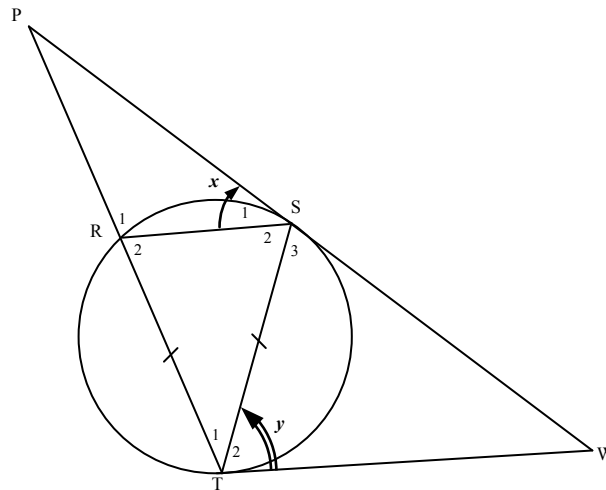
[10]

QUESTION/VRAAG 8



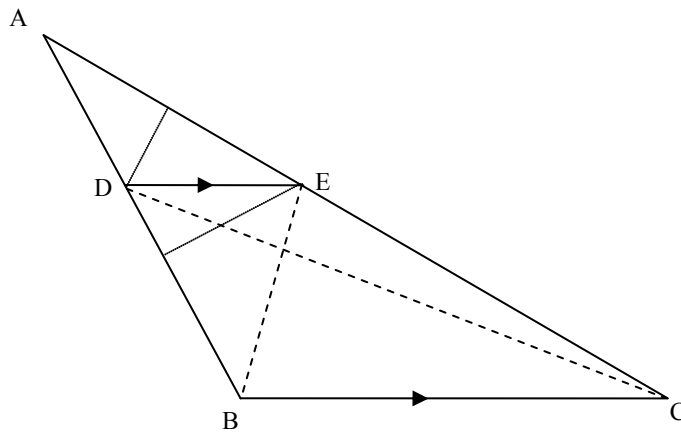
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| 8. | $\hat{T}_1 = 90^\circ$ (\angle s in a semi-circle)/(<i>\anglee in 'n halwe sirkel</i>) $PT = TW = 24$ cm (line from circ cent \perp ch) (<i>lyn van middelpunt \perp koord</i>) $OP^2 = OT^2 + PT^2$ (Pythagoras) $OP^2 = (10)^2 + (24)^2$ $OP^2 = 676$ $OP = 26$ cm Radius of smaller circle = 13 cm | ✓ $\hat{T}_1 = 90^\circ$ ✓ \angle s in a semi-circle ✓ line from circ cent \perp ch ✓ $OP = 26$ cm ✓ answer (5) |
|----|---|--|

QUESTION/VRAAG 9

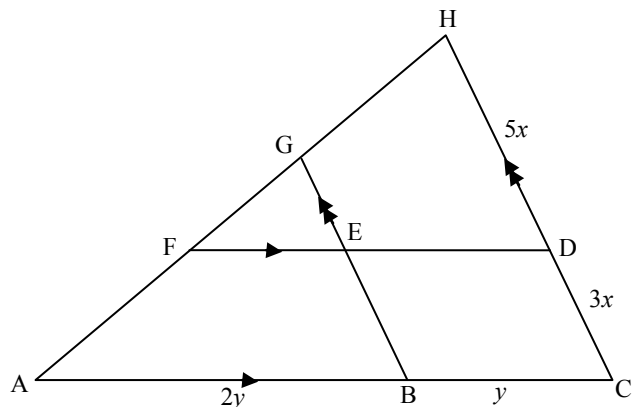


| | | |
|------------|--|---|
| <p>9.1</p> | <p>$\hat{R}_2 = y$ (tan ch th) (hoek tussen raaklyn en koord)</p> <p>$RT = ST$ (given)</p> <p>$\hat{S}_2 = y$ (\angles opp = sides)/(\anglee teenoor = sye)</p> <p>$SW = WT$ (tan from common point)/(<i>raaklyn vanaf selfde punt</i>)</p> <p>$\hat{S}_3 = y$ (\angles opp = sides)/(\anglee teenoor = sye)</p> <p>OR</p> <p>$SW = WT$ (tan from common point)/(<i>raaklyn vanaf selfde punt</i>)</p> <p>$\hat{S}_3 = y$ (\angles opp = sides)/(\anglee teenoor = sye)</p> <p>$\hat{R}_2 = \hat{S}_3 = y$ (tan ch th)/(hoek tussen raaklyn en koord)</p> <p>$RT = ST$ (given)</p> <p>$\hat{S}_2 = y$ (\angles opp = sides)/(\anglee teenoor = sye)</p> | <p>✓ $\hat{R}_2 = y$</p> <p>✓ tan ch th</p> <p>✓ $\hat{S}_2 = y$</p> <p>✓ \angles opp = sides</p> <p>✓ tan from common point</p> <p>✓ $\hat{S}_3 = y$</p> <p>(6)</p> <p>✓ tan from common point</p> <p>✓ $\hat{S}_3 = y$</p> <p>✓ $\hat{R}_2 = y$</p> <p>✓ tan ch th</p> <p>✓ $\hat{S}_2 = y$</p> <p>✓ \angles opp = sides</p> <p>(6)</p> |
| <p>9.2</p> | <p>In ΔPRS and ΔPST</p> <p>i. \hat{P} is common</p> <p>ii. $\hat{T}_1 = \hat{S}_1 = x$ (tan ch th)/ (hoek tussen raaklyn en koord)</p> <p>iii. $\hat{R}_1 = \hat{PST} = x + y$ (3^{rd} \angle of the Δ)</p> <p>$\Delta PRS \parallel \Delta PST$ ($\angle\angle\angle$)</p> | <p>✓ \hat{P} is common</p> <p>✓ $\hat{T}_1 = \hat{S}_1 = x$</p> <p>✓ $\hat{R}_1 = \hat{PST} = x + y$ OR ($\angle\angle\angle$)</p> <p>(3)</p> |
| <p>9.3</p> | <p>$\frac{PS}{PT} = \frac{RS}{ST}$ ($\parallel \Delta$s)</p> <p>$ST = RT$ (given)</p> <p>$\frac{PS}{PT} = \frac{RS}{RT}$</p> <p>$PS \times RT = RS \times PT$</p> | <p>✓ $\frac{PS}{PT} = \frac{RS}{ST}$</p> <p>✓ $\parallel \Delta$s</p> <p>✓ $ST = RT$</p> <p>(3)</p> <p>[12]</p> |

QUESTION 10



| | | |
|------|---|--|
| 10.1 | <p>Join E to B and D to C.</p> $\frac{\text{area } \triangle AED}{\text{area } \triangle DEB} = \frac{AD}{DB} \quad (\text{common altitudes})/$ <p style="text-align: center;"><i>(gemeenskaplike hoogtelyne)</i></p> $\frac{\text{area } \triangle AED}{\text{area } \triangle DEC} = \frac{AD}{EC} \quad (\text{common altitudes})$ <p>area $\triangle AED$ is common</p> <p>area $\triangle DEB = \text{area } \triangle DEC$ (DE \parallel BC; same base BC)</p> $\frac{\text{area } \triangle AED}{\text{area } \triangle DEB} = \frac{\text{area } \triangle AED}{\text{area } \triangle DEC}$ $\frac{AD}{DB} = \frac{AD}{EC}$ | <p>✓ construction</p> $\checkmark \frac{\text{area } \triangle AED}{\text{area } \triangle DEB} = \frac{AD}{DB}$ <p>✓ = alts \therefore ratio areas = ratio bases</p> $\checkmark \frac{\text{area } \triangle AED}{\text{area } \triangle DEC} = \frac{AD}{EC}$ <p>✓ area $\triangle DEB = \text{area } \triangle DEC$ and</p> <p>✓ DE \parallel BC; same base BC</p> $\checkmark \frac{\text{area } \triangle AED}{\text{area } \triangle DEB} = \frac{\text{area } \triangle AED}{\text{area } \triangle DEC}$ <p style="text-align: right;">(7)</p> |
|------|---|--|



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| 10.2.1 | $\frac{HF}{AH} = \frac{5}{8} \quad (\text{FD} \parallel \text{AC}; \text{Prop Th/Verhouding St})$ $HF = \frac{5}{8} AH$ $HF = \frac{5}{8} (48)$ $HF = 30 \text{ cm}$ | $\checkmark \frac{HF}{AH} = \frac{5}{8}$ $\checkmark \text{FD} \parallel \text{AC}$ $\checkmark \text{answer}$ <p style="text-align: right;">(3)</p> |
| 10.2.2 | $AF = 18 \text{ cm}$ $\frac{AF + FG}{HF - FG} = \frac{2}{1} \quad (\text{BG} \parallel \text{CH}; \text{Prop Th/Verhouding St})$ $\frac{18 + FG}{30 - FG} = 2$ $18 + FG = 2(30 - FG)$ $18 + FG = 60 - 2FG$ $3FG = 42$ $FG = 14 \text{ cm}$ <p>OR</p> $\frac{GH}{AH} = \frac{BC}{AC} \quad (\text{BG} \parallel \text{CH}; \text{Prop Th/Verhouding St})$ $\frac{GH}{48} = \frac{1}{3}$ $GH = 16 \text{ cm}$ $FG = HF - GH$ $= 30 - 16$ $= 14 \text{ cm}$ | $\checkmark AF = 18$ $\checkmark \frac{AF + FG}{HF - FG} = \frac{2}{1}$ $\checkmark \text{answer}$ $\checkmark \frac{GH}{AH} = \frac{BC}{AC}$ $\checkmark GH = 16 \text{ cm}$ $\checkmark \text{answer}$ <p style="text-align: right;">(3)</p> |
| 10.2.3 | $EF : ED = GF : GH \quad (\text{BG} \parallel \text{CH}; \text{Prop Th/Verhouding St})$ $EF : ED = 14 : 16$ $= 7 : 8$ | $\checkmark EF : ED = GF : GH$ $\checkmark \text{answer}$ <p style="text-align: right;">(2)</p> |

[15]

TOTAL/TOTAAL: 100