basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

MODEL 2014

MEMORANDUM

PUNTE: 150

Hierdie memorandum bestaan uit 13 bladsye.

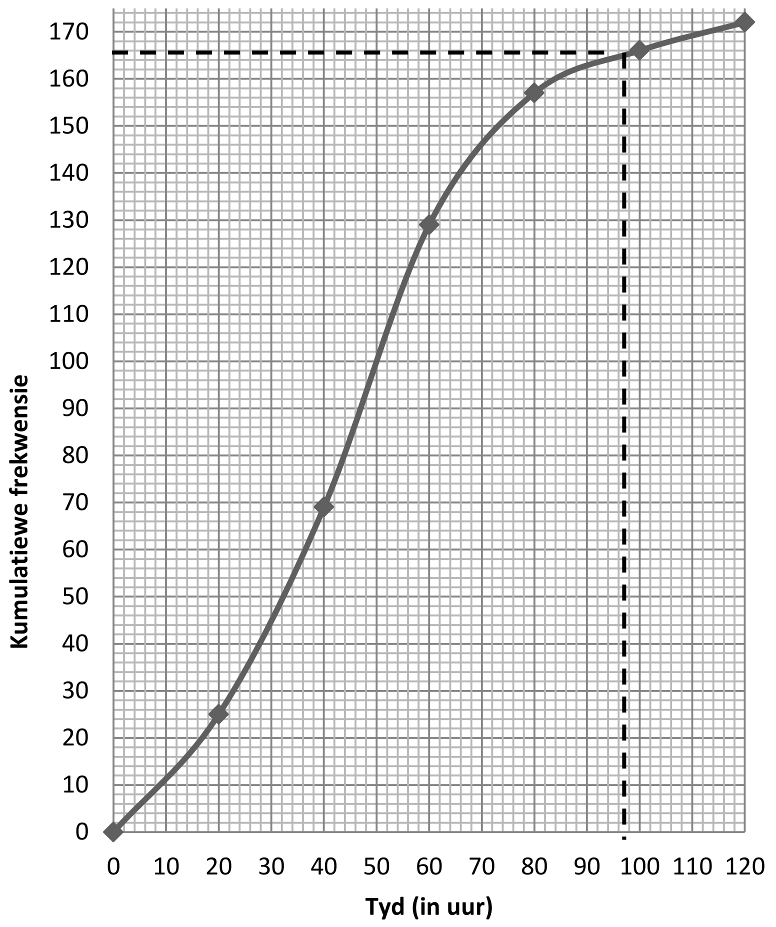
LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.*
- *Indien 'n kandidaat 'n poging om 'n vraag te beantwoord gekanselleer het en die vraag nie weer gedoen het nie, merk die gekanselleerde poging.*
- *Volgehoue akkuraatheid is van toepassing in **ALLE** aspekte van die nasien-memorandum.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, is ONAANVAARBAAR.*

VRAAG 1

1.1	Indien die aantal dae wat 'n atleet oefen toeneem, neem die tyd waarin die 100m naelloop afgelê word, af. **OF** Indien die aantal dae wat 'n atleet oefen afneem, neem die tyd waarin die 100m naelloop afgelê word, toe. **OF** Hoe meer dae 'n atleet oefen, hoe korter is die tyd wat hy die 100m naelloop aflê.	✓ verduideliking (1)
1.2	(60 ; 18,1)	✓ (1)
1.3	a = 17,81931464… b = – 0,070685358... $\therefore \hat{y} = -0,07x + 17,82$	✓✓ a ✓ b ✓ vergelyking (4)
1.4	$\therefore \hat{y} \approx -0,07(45) + 17,82$ $\approx 14,67$ sekondes	✓ substitusie ✓ antwoord (2)
1.5	r = – 0,74 (–0,740772594...)	✓✓ r (2)
1.6	Daar is 'n redelike sterk verwantskap tussen die veranderlikes.	✓ redelik sterk (1) **[11]**

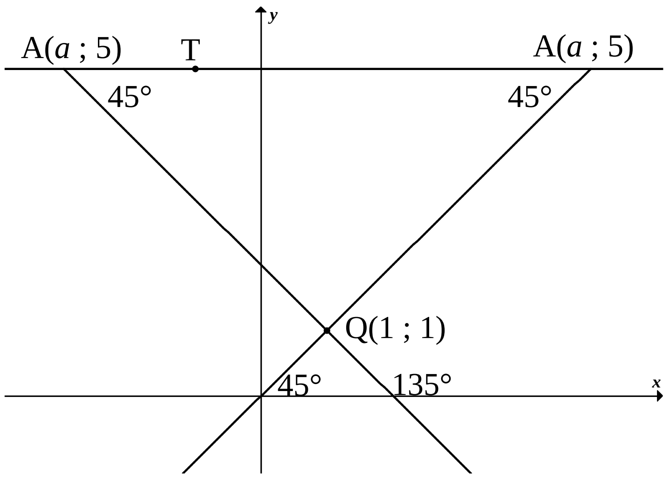
VRAAG 2

<table>
<tr><td>2.1</td><td>
</td><td>✓anker by 0
✓plot by boonste limiete
✓ gladde kurwe
(3)</td></tr>
<tr><td>2.2</td><td>$40 \leq t < 60$</td><td>✓ klas
(1)</td></tr>
<tr><td>2.3</td><td>(96 ; 164)
$\therefore$ 172 – 164 = 8 leerders</td><td>✓164
✓ 8
(2)</td></tr>
<tr><td>2.4</td><td>Frekwensie: 25; 44; 60; 28; 9; 6

$\text{gemiddelde} = \dfrac{25\times10+44\times30+60\times50+28\times70+9\times90+6\times110}{172}$
$= \dfrac{8000}{172}$
= 46,51 uur</td><td>✓frekwensie
✓ middelpunte
✓ $\dfrac{8000}{172}$
✓ antwoord
(4)
[10]</td></tr>
</table>

VRAAG 3

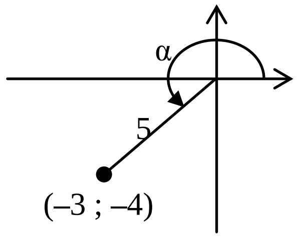
3.1	K(7 ; 0)	✓ antwoord (1)
3.2	$1=\frac{x_M+7}{2}$ *en* $1=\frac{y_M+3}{2}$ $\therefore$ M(– 5 ; – 1)	✓ x ✓ y (2)
3.3	$m_{PM}=\frac{3-1}{7-1}$ $=\frac{1}{3}$	✓ substitusie ✓antwoord (2)
3.4	$\tan\ P\hat{S}K=m_{PM}=\frac{1}{3}$ $P\hat{S}K=\tan^{-1}\left(\frac{1}{3}\right)=18{,}43°$ $\therefore\ \theta=180°-90°-18{,}43°=71{,}57°$	✓ $\tan P\hat{S}K=m_{PM}$ ✓ $P\hat{S}K$ ✓ θ (3)
3.5	$\cos 71{,}57°=\frac{PK}{PS}=\frac{3}{PS}$ $PS=\frac{3}{\cos 71{,}57°}$ = 9,49 eenhede **OF** $\sin 18{,}43°=\frac{PK}{PS}=\frac{3}{PS}$ $PS=\frac{3}{\sin 18{,}43°}$ = 9,49 eenhede	✓ korrekte verhouding ✓ PS onderwerp ✓ antwoord (3) ✓ korrekte verhouding ✓ PS onderwerp ✓ antwoord (3)
3.6	N(x ; –2x + 17) $m_{TN}=m_{PM}$ (TN | | PM) $\frac{-2x+17-5}{x-(-1)}=\frac{1}{3}$ $-6x+36=x+1$ $-7x=-35$ $x=5$ $\therefore\ y=-2(5)+17=7$ $\therefore$ N(5 ; 7) **OF**	✓ N in terme van x ✓ gelyke gradiënte ✓ substitusie ✓ x -waarde ✓ y -waarde (5)

	$m_{TM} = \frac{1}{3}$ (TN | | PM) vergelyking van TM: $y - y_1 = \frac{1}{3}(x - x_1)$ $y - 5 = \frac{1}{3}(x - (-1))$ $y - 5 = \frac{1}{3}x + \frac{1}{3}$ $y = \frac{1}{3}x + 5\frac{1}{3}$ **OF** $y = \frac{1}{3}x + c$ $5 = \frac{1}{3}(-1) + c$ $5\frac{1}{3} = c$ $y = \frac{1}{3}x + 5\frac{1}{3}$ $-2x + 17 = \frac{1}{3}x + 5\frac{1}{3}$ $-2\frac{1}{3}x = -11\frac{2}{3}$ $x = 5$ $\therefore y = -2(5) + 17 = 7$ $\therefore$ N(5 ; 7)	✓ m_{TM} ✓ vergelyking van TM ✓ stel gelyk aan mekaar ✓ x -waarde ✓ y -waarde (5)
3.7.1	$y = 5$	✓ vergelyking (1)
3.7.2	 gradiënt van AQ = tan 45° of tan 135° = 1 of –1 $m_{AQ} = \frac{5-1}{a-1} = \pm 1$ $\therefore$ a – 1 = 4 of –4 $\therefore$ a = 5 of –3	✓ m_{AQ} = 1 of ✓ m_{AQ} = –1 ✓ substitusie in gradiëntformule ✓ x-waarde ✓ y-waarde (5) **[22]**

VRAAG 4

4.1	M(–1 ; –1)	✓ antwoord (1)
4.2	$m_{NT} = \frac{2-1}{3-4} = -1$ $\therefore\ m_{AT} = 1$ (radius ⊥ raaklyn) $y - 1 = 1(x-4)$ $y = x - 3$	✓ m_{NT} ✓ m_{AT} ✓ rede ✓ substitusie van *m* en (4 ; 1) ✓ vergelyking (5)
4.3	MR ⊥ AB (lyn vanaf midpt na midpt van koord) $MB^2 = MR^2 + RB^2$ (Stelling van Pythagoras) $9 = (\frac{\sqrt{10}}{2})^2 + RB^2$ $RB^2 = \frac{13}{2}$ $RB = \sqrt{\frac{13}{2}}$ $AB = 2\left(\sqrt{\frac{13}{2}}\right) = \sqrt{26}$ eenhede	✓ MR ⊥ AB ✓MB = 3 ✓ substitusie in stelling van Pythagoras ✓ AB in wortelvorm (4)
4.4	$MN^2 = (-1-3)^2 + (-1-2)^2$ $= 16 + 9$ $= 25$ MN = 5 eenhede	✓ substitusie in afstandformule ✓ antwoord (2)
4.5	$r = 5 - 3 = 2$ eenhede $\therefore\ (x-3)^2 + (y-2)^2 = 4$ $\therefore\ x^2 + y^2 - 6x - 4y + 9 = 0$	✓r ✓substitusie in sirkelvergelyking ✓vergelyking (3) **[15]**

VRAAG 5

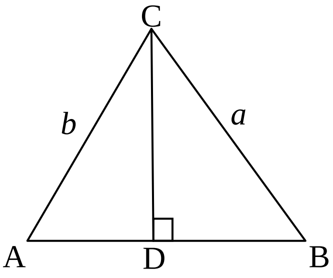
5.1.1	$-\sin\alpha$ $= -(-\frac{4}{5}) = \frac{4}{5}$	✓ reduksie ✓ antwoord (2)
5.1.2	$(-4)^2 + b^2 = 5^2$ $b^2 = 25 - 16 = 9$ $b = -3$ $\cos\alpha = \frac{-3}{5}$  	✓ $b = -3$ ✓antwoord (2)
5.1.3	$\sin(\alpha - 45°)$ $= \sin\alpha\cos 45° - \cos\alpha\sin 45°$ $= -\frac{4}{5}\cdot\frac{1}{\sqrt{2}} - (-\frac{3}{5})\cdot\frac{1}{\sqrt{2}}$ $= -\frac{1}{5\sqrt{2}}$ **OF** $\sin(\alpha - 45°)$ $= \sin\alpha\cos 45° - \cos\alpha\sin 45°$ $= -\frac{4}{5}\cdot\frac{\sqrt{2}}{2} - (-\frac{3}{5})\cdot\frac{\sqrt{2}}{2}$ $= -\frac{\sqrt{2}}{10}$	✓ uitbreiding ✓ $\frac{1}{\sqrt{2}}$ ✓ antwoord in eenvoudigste vorm (3) ✓ uitbreiding ✓ $\frac{\sqrt{2}}{2}$ ✓ antwoord in eenvoudigste vorm (3)
5.2.1	$LHS = \frac{8\sin x.\cos x}{\sin^2 x - \cos^2 x}$ $= \frac{4(2\sin x.\cos x)}{\sin^2 x - \cos^2 x}$ $= \frac{4\sin 2x}{-(\cos^2 x - \sin^2 x)}$ $= \frac{4\sin 2x}{-\cos 2x}$ $= -4\tan 2x$	✓ sin x ✓ cos x ✓ $\cos^2 x$ ✓ 4 sin 2x ✓ faktoriseer ✓ – cos 2x (6)
5.2.2	Ongedefinieer as cos 2x = 0 of tan 2x = ∞: x = 45° en x = 135°	✓ 45° ✓ 135° (2)

5.3	$1-2\sin^2\theta+4\sin^2\theta-5\sin\theta-4=0$ $2\sin^2\theta-5\sin\theta-3=0$ $(2\sin\theta+1)(\sin\theta-3)=0$ $\therefore\ \sin\theta=-\frac{1}{2}$ of $\sin\theta=3$ (geen oplossing) $\therefore\theta=210°+360°k$ of $\theta=330°+360°k\ ;\ k\in Z$ **OF** $\therefore\theta=210°+360°k$ of $\theta=30°+360°k\ ;\ k\in Z$	✓ $1-2\sin^2\theta$ ✓ standaardvorm ✓ faktore ✓ geen oplossing ✓ 210° ✓ 330° ✓ $+360°k\ ;\ k\in Z$ (7) **[22]**

VRAAG 6

6.1	$b=\frac{1}{2}$	✓ waarde van *b* (1)
6.2	A(30° ; 1)	✓ 30° ✓ 1 (2)
6.3	$x=160°$	✓ $x=160°$ (1)
6.4	$h(x)=2\cos(x-30°)+1$ $y\in[-1\ ;\ 3]$ **OF** $-1\le y\le 3$	✓ kritiese waardes ✓ notasie (2) **[6]**

VRAAG 7

7.1	Trek CD ⊥ AB In ΔACD: $\sin A = \frac{CD}{b}$ $\therefore$ CD = b. sin A  In ΔCBD: $\sin B = \frac{CD}{a}$ $\therefore$ CD = a. sin B $\therefore$ b . sin A = a. sin B $\therefore \frac{\sin A}{a} = \frac{\sin B}{b}$	✓ konstruksie ✓ sin A ✓ maak CD die onderwerp ✓ sin B ✓ b. sin A = a. sin B (5)
7.2.1	$S\hat{P}Q = 180° - 2x$ (teenoorst ∠e van koordevierh) $P\hat{S}Q + P\hat{Q}S = 2x$ (som van ∠e in Δ) $P\hat{S}Q = P\hat{Q}S = x$ (∠e teenoor gelyke sye)	✓ $S\hat{P}Q = 180° - 2x$ (S/R) ✓ rede (2)
7.2.2	$\frac{\sin S\hat{P}Q}{SQ} = \frac{\sin P\hat{S}Q}{PQ}$ $\frac{\sin(180° - 2x)}{SQ} = \frac{\sin x}{k}$ $SQ = \frac{k \sin 2x}{\sin x}$ $SQ = \frac{k(2\sin x.\cos x)}{\sin x} = 2k\cos x$ **OF** $SQ^2 = PQ^2 + PS^2 - 2PQ.PS.\cos S\hat{P}Q$ $= k^2 + k^2 - 2.k.k.\cos(180° - 2x)$ $= 2k^2 + 2k^2\cos 2x$ $= 2k^2 + 2k^2(2\cos^2 x - 1)$ $= 4k^2\cos^2 x$ $SQ = 2k\cos x$	✓ substitusie in korrekte formule ✓ sin 2x ✓ SQ onderwerp ✓ $2\sin x.\cos x$ (4) ✓ substitusie in korrekte formule ✓ – cos 2x ✓ $2\cos^2 x - 1$ ✓ vereenvoudig (4)
7.2.3	$\tan y = \frac{3}{k}$ $k = \frac{3}{\tan y}$ $\therefore$ $SQ = 2\cos x\left(\frac{3}{\tan y}\right)$ $= \frac{6\cos x}{\tan y}$	✓ tan-verhouding ✓ k onderwerp en substitusie (2) **[13]**

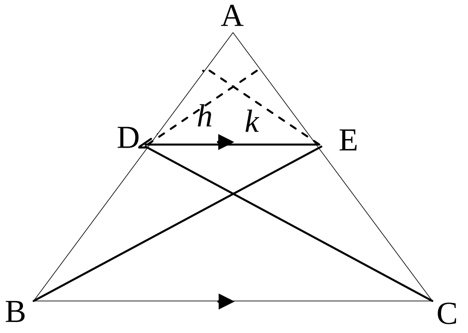
VRAAG 8

8.1	die hoek onderspan in die teenoorstaande sirkelsegment	✓korrekte stelling (1)
8.2.1	$\hat{B}_1 = \hat{E}_1 = 68°$ (rkl-koordst)	✓ $\hat{E}_1 = 68°$ ✓ rede (2)
8.2.2	$\hat{E}_1 = \hat{B}_3 = 68°$ (verwiss∠e; AE | | BC)	✓ $\hat{B}_3 = 68°$ (S/R) (1)
8.2.3	$\hat{D}_1 = \hat{B}_3 = 68°$ (buite∠ v koordevh)	✓ $\hat{D}_1 = 68°$ ✓ rede (2)
8.2.4	$\hat{E}_2 = 20° + 68°$ (buite∠ v Δ) $= 88°$	✓ $\hat{E}_2 = 88°$ (S/R) (1)
8.2.5	$\hat{C} = 180° - 88°$ (tos ∠e v koordevh) $= 92°$	✓ $\hat{C} = 92°$ ✓ rede (2) **[9]**

VRAAG 9

9.1	$\hat{D}_4 = \hat{A} = x$ (rkl-koordstelling) $\hat{A} = \hat{D}_2 = x$ ($\angle$e tos gelyke sye)	✓ $\hat{A} = x$ ✓ rede ✓ $\hat{A} = \hat{D}_2 = x$ (S/R) (3)
9.2	$\hat{M}_1 = 2x$ (buite $\angle$vΔ) OF ($\angle$ by midpt = 2$\angle$ by omtr) $M\hat{D}E = 90°$ (radius $\perp$ rkl) $\hat{M}_2 = 90° - 2x$ $\therefore \hat{E} = 180° - (90° + 90° - 2x)$ (som v $\angle$e in ΔMDE) $= 2x$ $\therefore$ CM is 'n rkl (omgek rkl-koordst)	✓ $\hat{M}_1 = 2x$ (S/R) ✓ $M\hat{D}E = 90°$ (S/R) ✓ $\hat{E} = 2x$ ✓ rede (4)
9.3	$\hat{M}_3 = 90°$ (EM $\perp$ AC) $A\hat{D}B = 90°$ ($\angle$ in halfsirkel) $\therefore$ FMBD is koordevh (buite$\angle$ v vh = tos binne $\angle$) **OF** $E\hat{M}C = 90°$ (EM $\perp$ AC) $A\hat{D}B = 90°$ ($\angle$ in halfsirkel) $\therefore$ FMBD is koordevh (tos $\angle$e v vh suppl)	✓ $\hat{M}_3 = 90°$ ✓ $A\hat{D}B = 90°$ (S/R) ✓ rede (3) ✓ $E\hat{M}C = 90°$ ✓ $A\hat{D}B = 90°$ (S/R) ✓ rede (3)
9.4	$DC^2 = MC^2 - MD^2$ (Pythagoras) $= (3BC)^2 - (2BC)^2$ (MB = MD = radii) $= 9BC^2 - 4BC^2$ $= 5BC^2$	✓ Pythagoras ✓ substitusie ✓ $9BC^2 - 4BC^2$ (3)
9.5	In ΔDBC en ΔDFM: $\hat{D}_4 = \hat{D}_2 = x$ (bewys in 9.1) $\hat{B}_1 = \hat{F}_2$ (buite $\angle$ v koordevh) $\hat{C} = \hat{M}_2$ $\therefore \Delta DBC \mid\mid\mid \Delta DFM$ ($\angle$; $\angle$; $\angle$)	✓ $\hat{D}_4 = \hat{D}_2$ ✓ $\hat{B}_1 = \hat{F}_2$ ✓ rede ✓ $\hat{C} = \hat{M}_2$ of ($\angle$; $\angle$; $\angle$) (4)
9.6	$\frac{DM}{FM} = \frac{DC}{BC}$ (ΔDBC ||| ΔDFM) $= \frac{\sqrt{5}BC}{BC}$ $= \sqrt{5}$	✓ S ✓ antwoord (2) **[19]**

VRAAG 10

10.1	 Konstruksie: Verbind DC en BE en trek hoogtes *k* en *h* $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEB}} = \frac{\frac{1}{2}.\text{AD}.k}{\frac{1}{2}.\text{DB}.k} = \frac{\text{AD}}{\text{DB}}$ (gelyke hoogtes) $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEC}} = \frac{\frac{1}{2}.\text{AE}.h}{\frac{1}{2}.\text{E}C.h} = \frac{\text{AE}}{\text{EC}}$ (gelyke hoogtes) Maar Opp ΔDEB = Opp ΔDEC (dies basis, dies hoogte) $\therefore \frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEB}} = \frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEC}}$ $\therefore \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$	✓ konstruksie ✓ $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEB}} = \frac{\text{AD}}{\text{DB}}$ ✓ rede ✓ $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEC}} = \frac{\text{AE}}{\text{EC}}$ ✓ Area ΔDEB = Area ΔDEC (S/R) ✓ $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEB}} = \frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEC}}$ (6)

10.2.1	$\frac{AB}{BE}=\frac{AC}{CD}$ (Ewered st; BC | | ED) $\frac{1}{3}=\frac{3}{CD}$ $\therefore$ CD = 9 eenhede	✓ $\frac{AB}{BE}=\frac{AC}{CD}$ (S/R) ✓ substitusie ✓ antwoord (3)
10.2.2	$\frac{DG}{GA}=\frac{FD}{FE}$ (Ewered st; FG | | EA) $\frac{9-x}{3+x}=\frac{3}{6}$ $54-6x=9+3x$ $-9x=-45$ $x=5$	✓ $\frac{DG}{GA}=\frac{FD}{FE}$ (S/R) ✓ substitusie ✓ vereenvoudig ✓ antwoord (4)
10.2.3	In ΔABC en ΔAED: $\hat{A}$ is gemeen $A\hat{B}C=\hat{E}$ (ooreenk ∠s; BC | | ED) $A\hat{C}B=\hat{D}$ (ooreenk ∠s; BC | | ED) ΔABC | | | ΔAED (∠, ∠, ∠) $\therefore \frac{BC}{ED}=\frac{AC}{AD}$ $\frac{BC}{9}=\frac{3}{12}$ $BC=2\frac{1}{4}$ eenhede	✓ $\hat{A}$ is gemeen ✓ $A\hat{B}C=\hat{E}$ (S/R) ✓ $A\hat{C}B=\hat{D}$ (S/R) of (∠; ∠; ∠) ✓ $\frac{BC}{ED}=\frac{AC}{AD}$ ✓ antwoord (5)
10.2.4	$\frac{\text{opp } \Delta ABC}{\text{opp } \Delta GFD}=\frac{\frac{1}{2}AC.BC.\sin A\hat{C}B}{\frac{1}{2}GD.FD.\sin \hat{D}}$ $=\frac{\frac{1}{2}(3)(2\frac{1}{4})\sin \hat{D}}{\frac{1}{2}(4)(3)\sin \hat{D}}$ (ooreenk ∠s; BC | | ED) $=\frac{9}{16}$	✓ gebruik v opp reël ✓ korrekte sye en ∠e ✓ substitusie v waardes ✓ $\sin A\hat{C}B=\sin \hat{D}$ (S/R) ✓ antwoord (5) **[23]**

TOTAAL: 150