basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

EXEMPLAR 2014

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

5. Answers only will not necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 4x = 0$ (2)

1.1.2 $x - 6 + \frac{2}{x} = 0$; $x \neq 0$. (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^{\frac{2}{3}} = 4$ (2)

1.1.4 $3^x(x-5) < 0$ (2)

1.2 Solve for x and y simultaneously:
$y = x^2 - x - 6$ **and** $2x - y = 2$ (6)

1.3 Simplify, without the use of a calculator:
$\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$ (3)

1.4 Given: $f(x) = 3(x-1)^2 + 5$ and $g(x) = 3$

1.4.1 Is it possible for $f(x) = g(x)$? Give a reason for your answer. (2)

1.4.2 Determine the value(s) of k for which $f(x) = g(x) + k$ has TWO unequal real roots. (2)
[23]

QUESTION 2

2.1 Given the arithmetic series: 18 + 24 + 30 + … + 300

2.1.1 Determine the number of terms in this series. (3)

2.1.2 Calculate the sum of this series. (2)

2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)

2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.

2.2.1 Determine the n^{th} term of the sequence. (2)

2.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)

2.2.3 Calculate the sum to infinity of this sequence. (2)
[16]

QUESTION 3

3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1.
The general term of the first differences is given by $4n + 6$.

3.1.1 Determine the value of a. (2)

3.1.2 Determine the formula for T_n. (4)

3.2 Given the series: $(1\times 2)+(5\times 6)+(9\times 10)+(13\times 14)+...+(81\times 82)$

Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4)
[10]

QUESTION 4

4.1 Given: $f(x)=\dfrac{2}{x+1}-3$

4.1.1 Calculate the coordinates of the y-intercept of f. (2)

4.1.2 Calculate the coordinates of the x-intercept of f. (2)

4.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (3)

4.1.4 One of the axes of symmetry of f is a decreasing function. Write down the equation of this axis of symmetry. (2)

4.2 The graph of an increasing exponential function with equation $f(x)=a.b^x+q$ has the following properties:

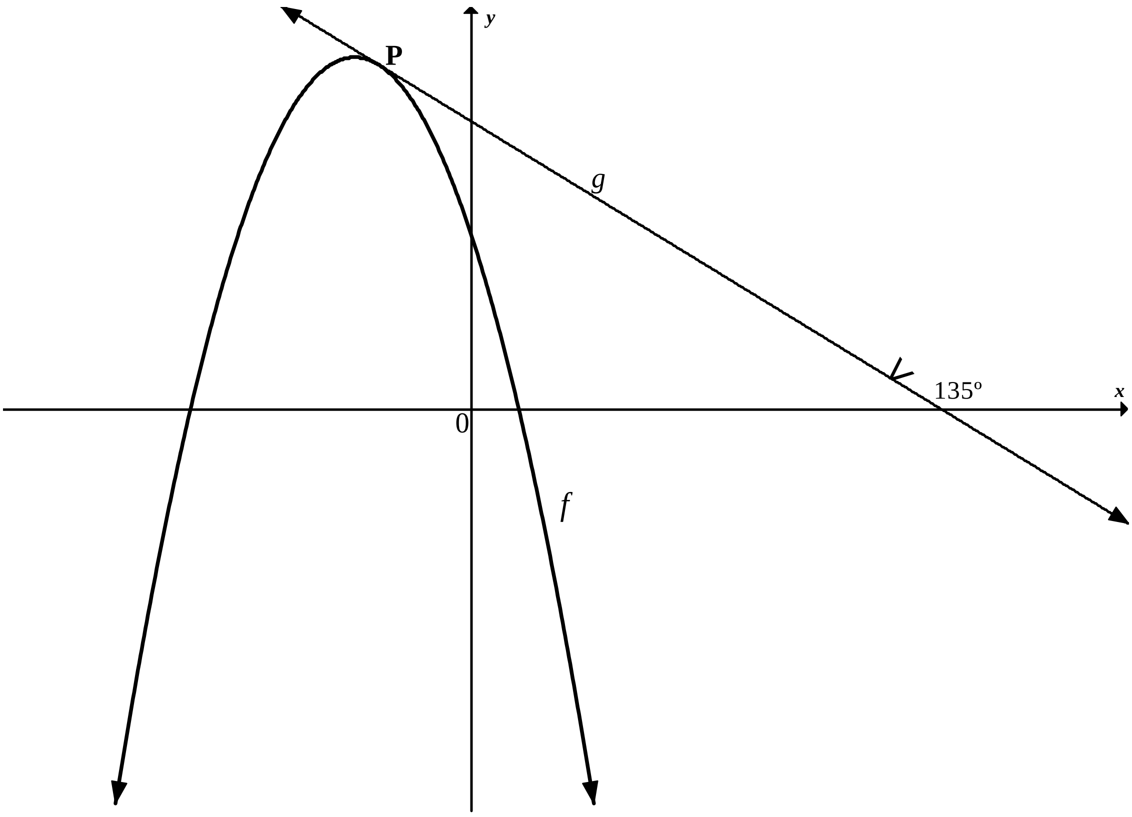
- Range: $y > -3$
- The points (0 ; – 2) and (1; – 1) lie on the graph of f.

4.2.1 Determine the equation that defines f. (4)

4.2.2 Describe the transformation from $f(x)$ to $h(x)=2.2^x+1$ (2)
[15]

QUESTION 5

The sketch below shows the graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$. The angle of inclination of graph g is 135° in the direction of the positive x-axis. P is the point of intersection of f and g such that g is a tangent to the graph of f at P.



5.1	Calculate the coordinates of the turning point of the graph of f.	(3)
5.2	Calculate the coordinates of P, the point of contact between f and g.	(4)
5.3	Hence or otherwise, determine the equation of g.	(2)
5.4	Determine the values of d for which the line $k(x) = -x + d$ will not intersect the graph of f.	(1)

[10]

QUESTION 6

The graph of g is defined by the equation $g(x)=\sqrt{ax}$. The point (8 ; 4) lies on g.

6.1 Calculate the value of a. (2)

6.2 If $g(x)>0$, for what values of x will g be defined? (1)

6.3 Determine the range of g. (1)

6.4 Write down the equation of g^{-1}, the inverse of g, in the form $y = ...$ (2)

6.5 If $h(x)=x-4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of h and g. (4)

6.6 Hence, or otherwise, determine the values of x for which $g(x)>h(x)$. (2)
[12]

QUESTION 7

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.1 Determine the selling price of the house. (1)

7.2 The period of the loan is 20 years and she starts repaying the loan one month after it was granted. Calculate her monthly instalment. (4)

7.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2)

7.4 Calculate the balance of her loan immediately after her 85^{th} instalment. (3)

7.5 She experienced financial difficulties after the 85^{th} instalment and did not pay any instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi owes on her bond at the end of the 89^{th} month. (2)

7.6 She decides to increase her payments to R8 500 per month from the end of the 90^{th} month. How many months will it take to repay her bond after the new payment of R8 500 per month? (4)
[16]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=3x^2-2$. (5)

8.2 Determine $\dfrac{dy}{dx}$ if $y=2x^{-4}-\dfrac{x}{5}$. (2)
[7]

QUESTION 9

Given: $f(x) = x^3 - 4x^2 - 11x + 30$.

9.1 Use the fact that $f(2) = 0$ to write down a factor of $f(x)$. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (4)

9.3 Calculate the coordinates of the stationary points of f. (5)

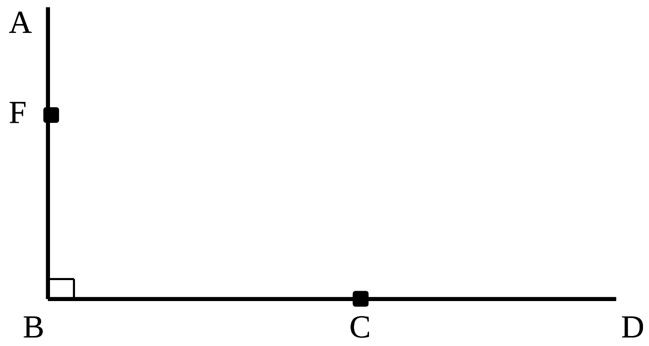
9.4 Sketch the curve of f in your ANSWER BOOK. Show all intercepts with the axes and turning points clearly. (3)

9.5 For which value(s) of x will $f'(x) < 0$? (2)

[15]

QUESTION 10

Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



10.1 Determine the distance between F and C in terms of t. (4)

10.2 After how long will the two cyclists be closest to each other? (4)

10.3 What will the distance between the cyclists be at the time determined in QUESTION 10.2? (2)

[10]

QUESTION 11

11.1 Events A and B are mutually exclusive. It is given that:

- P(B) = 2P(A)
- P(A or B) = 0,57

Calculate P(B). (3)

11.2 Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls. Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.

11.2.1 Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (4)

11.2.2 What is the probability that a yellow ball will be chosen from Bag A? (1)

11.2.3 What is the probability that a pink ball will be chosen? (3)
[11]

QUESTION 12

Consider the word M A T H S.

12.1 How many different 5-letter arrangements can be made using all the above letters? (2)

12.2 Determine the probability that the letters S and T will always be the first two letters of the arrangements in QUESTION 12.1. (3)
[5]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$ **;** $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$ **;** $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$