basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

FEBRUARY/MARCH 2014

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, 1 diagram sheet and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. ONE diagram sheet for QUESTION 13 is attached at the end of this question paper. Write your centre number and examination number on this diagram sheet in the spaces provided and insert the diagram sheet inside the back cover of your ANSWER BOOK.
8. An information sheet with formulae is included at the end of this question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $x^2 - 2x - 35 = 0$ (3)

1.1.2 $x^2 - 16 \geq 0$ (4)

1.1.3 $9.2^{x-1} = 2.3^x$ (3)

1.2 Given: $f(x) = x^2 - 5x + c$

Determine the value of c if it is given that the solutions of $f(x) = 0$ are $\frac{5 \pm \sqrt{41}}{2}$. (3)

1.3 Solve for x and y if: $3^{x-10} = 3^{3x}$ and $y^2 + x = 20$. (5)
[18]

QUESTION 2

2.1 A geometric sequence has $T_3 = 20$ and $T_4 = 40$.

Determine:

2.1.1 The common ratio (1)

2.1.2 A formula for T_n (3)

2.2 The following sequence has the property that the sequence of numerators is arithmetic and the sequence of denominators is geometric:

$$\frac{2}{1}; \frac{-1}{5}; \frac{-4}{25}; ...$$

2.2.1 Write down the FOURTH term of the sequence. (1)

2.2.2 Determine a formula for the n^{th} term. (3)

2.2.3 Determine the 500^{th} term of the sequence. (2)

2.2.4 Which will be the first term of the sequence to have a NUMERATOR which is less than –59? (3)
[13]

QUESTION 3

3.1 Given the arithmetic sequence: $w-3$; $2w-4$; $23-w$

3.1.1 Determine the value of w. (2)

3.1.2 Write down the common difference of this sequence. (1)

3.2 The arithmetic sequence 4 ; 10 ; 16 ; ... is the sequence of first differences of a quadratic sequence with a first term equal to 3.

Determine the 50$^{\text{th}}$ term of the quadratic sequence. (5)
[8]

QUESTION 4

In a geometric series, the sum of the first n terms is given by $S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of this series is 10.

4.1 Calculate the value of p. (4)

4.2 Calculate the second term of the series. (4)
[8]

QUESTION 5

5.1 Draw the graphs of $x^2 + y^2 = 16$ and $x + y = 4$ on the same set of axes in your ANSWER BOOK. (4)

5.2 Write down the coordinates of the points of intersection of the two graphs. (2)
[6]

QUESTION 6

Consider: $f(x)=\dfrac{6}{x-2}+3$

6.1 Write down the equations of the asymptotes of the graph of *f*. (2)

6.2 Write down the domain of *f*. (1)

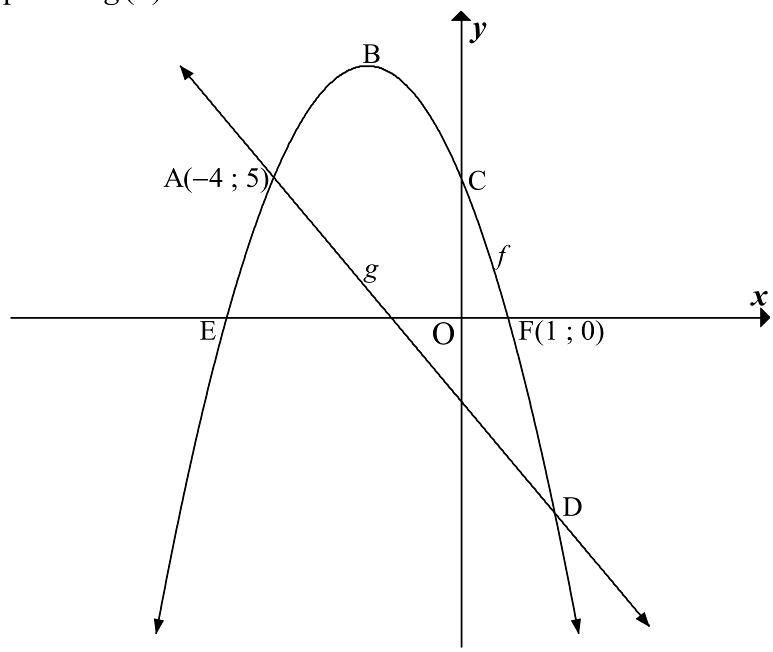
6.3 Draw a sketch graph of f in your ANSWER BOOK, indicating the intercept(s) with the axes and the asymptotes. (4)

6.4 The graph of f is translated to g. Describe the transformation in the form $(x\,;y)\rightarrow ...$ if the axes of symmetry of g are $y=x+3$ and $y=-x+1$. (4)

[11]

QUESTION 7

The graph of $f(x)=a(x-p)^2+q$ where *a*, *p* and *q* are constants, is given below.
Points E, F(1 ; 0) and C are its intercepts with the coordinates axes. A(– 4 ; 5) is the reflection of C across the axis of symmetry of *f*. D is a point on the graph such that the straight line through A and D has equation $g(x)=-2x-3$.



7.1 Write down the coordinates of C. (1)

7.2 Write down the equation of the axis of symmetry of *f*. (1)

7.3 Calculate the values of *a, p* and *q*. (6)

7.4 If $f(x)=-x^2-4x+5$, calculate the x-coordinate of D. (4)

7.5 The graph of f is reflected about the x-axis.

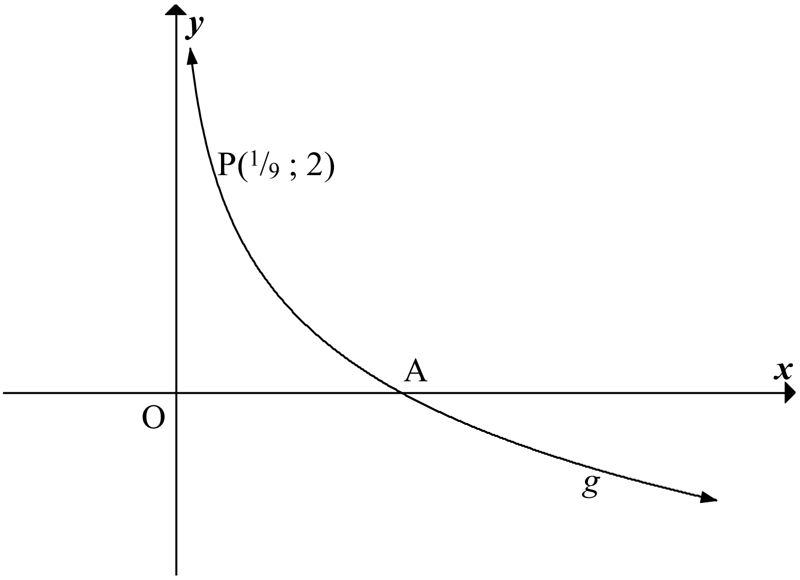
Write down the coordinates of the turning point of the new parabola. (2)

[14]

QUESTION 8

Given the graph of $g(x) = \log_{\frac{1}{3}} x$.

- A is the x-intercept of g.
- $P\left(\frac{1}{9}\,;2\right)$ is a point on g.



8.1 Write down the coordinates of A. (1)

8.2 Sketch the graph of g^{-1} indicating an intercept with the axes and ONE other point on the graph. (3)

8.3 Write down the domain of g^{-1}. (1)
[5]

QUESTION 9

Susan buys a car for R350 000. She secures a loan at an interest rate of 7% p.a., compounded monthly. The monthly instalment is R6 300. She pays the first instalment one month after the loan was secured.

9.1 Calculate the effective annual interest rate on the loan. Leave your answer correct to TWO decimal places. (3)

9.2 How many months will it take to repay the loan? (5)

9.3 Calculate the value of the final instalment. (5)

9.4 The value of the car depreciates at i % p.a. After 3 years its value is R252 000. Calculate i. (3)

[16]

QUESTION 10

10.1 Given: $f(x) = -\frac{2}{x}$

10.1.1 Determine $f'(x)$ from first principles. (5)

10.1.2 For which value(s) of x will $f'(x) > 0$? Justify your answer. (2)

10.2 Evaluate $\frac{dy}{dx}$ if $y = \frac{1}{4}x^2 - 2x$. (2)

10.3 Given: $y = 4\left(\sqrt[3]{x^2}\right)$ and $x = w^{-3}$

Determine $\frac{dy}{dw}$. (4)

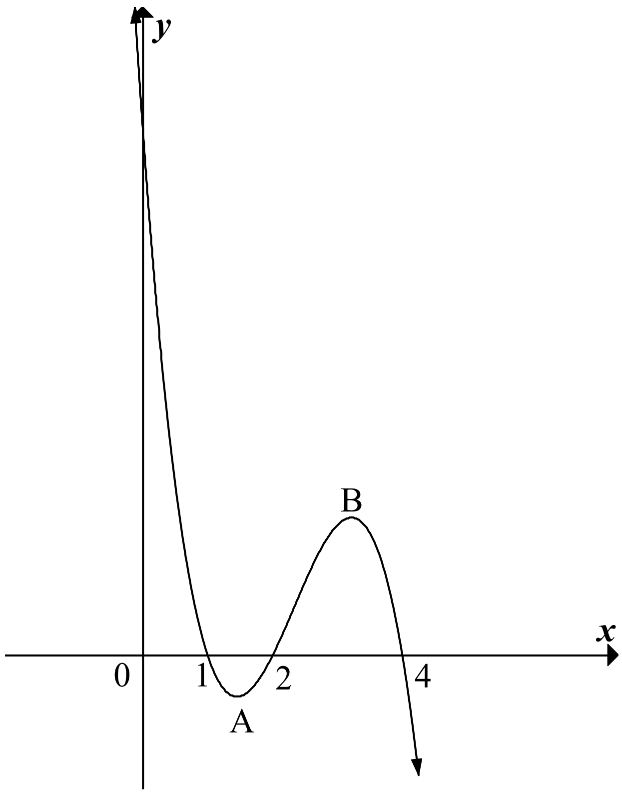
10.4 Given: $f(x) = ax^3 + bx^2 + cx + d$

Draw a possible sketch of $y = f'(x)$ if a, b and c are all NEGATIVE real numbers. (4)

[17]

QUESTION 11

The graph of $f(x) = -x^3 + ax^2 + bx + c$ is sketched below. The x-intercepts are indicated.



11.1 Calculate the values of a, b and c. (4)

11.2 Calculate the x-coordinates of A and B, the turning points of f. (5)

11.3 For which values of x will $f'(x) < 0$? (3)
[12]

QUESTION 12

A small business currently sells 40 watches per year. Each of the watches is sold at R144. For each yearly price increase of R4 per watch, there is a drop in sales of one watch per year.

12.1 How many watches are sold x years from now? (1)

12.2 Determine the annual income from the sale of watches in terms of x. (3)

12.3 In what year and at what price should the watches be sold in order for the business to obtain a maximum income from the sale of watches? (4)
[8]

QUESTION 13

A sweet factory produces two types of toffees, Taffy and Chewy, and stores them in containers.

- The quantities of butter and sugar (in kilograms) used in each container of toffies are as follows:
  - Taffy toffees contain 40 kg of butter and 64 kg of sugar for every container of toffees.
  - Chewy toffees contain 50 kg of butter and 40 kg of sugar for every container of toffees.
- Each week, the factory has a maximum of 2 000 kg of butter and 2 560 kg of sugar available.
- The factory must produce at least 15 containers of Taffy toffees per week.

Let x and y be the number of containers of Taffy and Chewy toffees produced each week, respectively.

13.1 Write down all the constraints which describe the production process above. (5)

13.2 Sketch the system of constraints (inequalities) on the graph paper on DIAGRAM SHEET 1, clearly indicating the feasible region. (4)

13.3 Write down the maximum number of containers of Taffy toffees that can be produced under these conditions. (2)

13.4 If the profit earned per week by the factory is R1 400 per container of Taffy toffees and R1 000 per container of Chewy toffees, what amount of each type of toffee needs to be produced in order to make a maximum profit per week? (3)

[14]

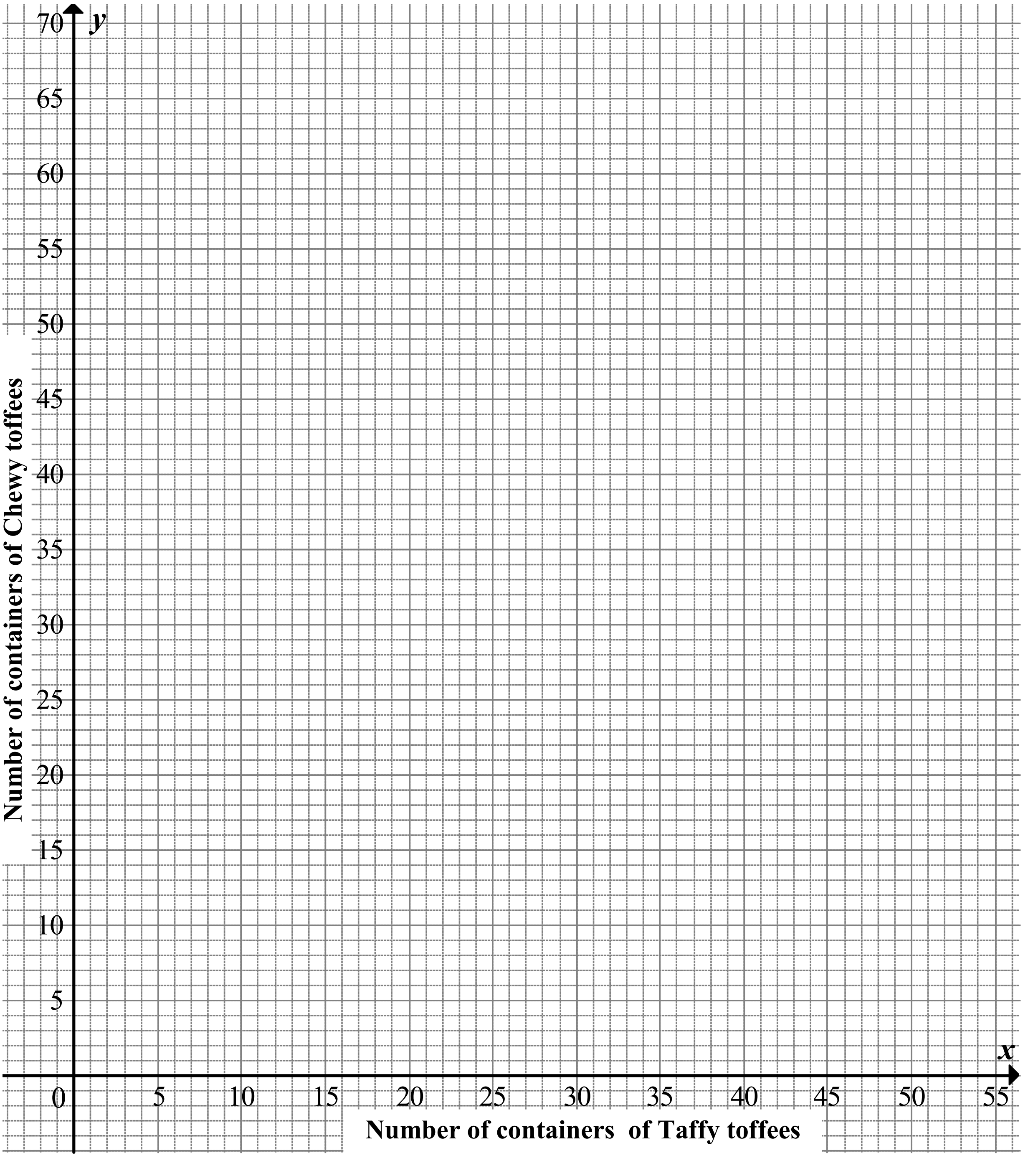
TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 13.2



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x\ ;\ y) \rightarrow (x\cos\theta - y\sin\theta\ ;\ y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$